Excellence in mathematics education: Models for teacher education practices

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This paper provides an overview of my research projects that are part of collaborative research program seeking to illustrate the complexity of excellence in mathematics education. The aim of the first two projects were to the theoretical and empirical grounding for an innovative approach in differential equations called the *Inquiry Oriented Differential Equations* (IO-DE) project. The aim of the third project was to provide a model of professional development of mathematics teachers in South Korea.

The concept of excellence in mathematics education is one of the most elusive in the educational literature. Writers often use the term *excellence* and assume their readers know what it means. Dictionaries give such definitions as "the quality of being excellent", "an excellent or valuable quality", and the "quality of being outstanding or extremely good". Thus, arriving at a simple definition is a challenging matter. However, excellence in mathematics education can be described in terms of research-based curriculum development, research-based teaching practices, and professional development. I would like to describe my research project in related to three aspects of excellence in mathematics education.

Inquiry Oriented Differential Equations (IO-DE) project

The *Inquiry-Orientated Differential Equations* (IO-DE) project is an example of a collaborative effort between mathematics educators and mathematicians that seeks to explore the prospects and possibilities for improving undergraduate mathematics education, using differential equations as a case example (Kwon, 2002). In this section, I highlight the theoretical background for the IO-DE project and a summary of quantitative and qualitative studies of the IO-DE project on student learning and how teachers create and sustain an inquiry-oriented learning environment.

While there are clear calls for inquiry in both science and mathematics classrooms, what exactly characterizes an inquiry-oriented classroom is less clear. To clarify the nature of inquiry-oriented classrooms and to provide a more comprehensive perspective on the complexity of teaching and learning, Rasmussen and Kwon (2007) characterize inquiry in terms of both student activity and teacher activity. In particular, students learn new mathematics by inquiry, which involves solving novel problems, debating mathematical solutions, posing and following up on conjectures, and explaining and justifying one's thinking. The first function that student inquiry serves is to learn new mathematics by engaging in genuine argumentation. The second function that student inquiry serves is to empower learners to see themselves as capable of re-inventing mathematics and to see mathematics itself as a human activity. On the other hand, teachers also engage in inquiry. Teacher inquiry centres on inquiring into their students' mathematical thinking and reasoning. Teacher inquiry into student thinking serves three functions. First, it enables teachers to interpret how their students build mathematical ideas. Second, it provides an opportunity for teachers to learn something new about particular mathematical ideas in light of student thinking. Third, it better positions teachers to follow up on students' thinking by posing new questions and tasks.

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Accomplishing these three goals was facilitated by conducting research in three related strands: (1) adaptation of an innovative instructional design approach at the undergraduate level; (2) systematic study of student thinking as they build ideas and teacher knowledge to support students' re-invention; and (3) careful attention to the social production of meaning and student identity. These three strands do not represent a linear progression in our research. We conducted research in these three strands concurrently and view the strands as complementary.

The implications of the IO-DE project are threefold. First, based on the results of the pre-test and the delayed post-test (Kwon, 2005; Rasmussen et al., 2006), the IO-DE students from each of the four institutions outperformed traditionally taught comparison students on the post-test. This result was true for both males and females and for high and low achieving students. This result demonstrates that this instructional approach can be applicable to university mathematics. Secondly and more importantly, the instructional methods and curriculum design approach guided by Realistic Mathematics Education (RME) framework are applicable to promoting student learning in all mathematics classrooms (Kwon, 2002). Thirdly, the IO-DE project can provide a model for how it is that teachers create and sustain inquiry-oriented learning environments in which students gain mathematical power and sophistication.

Since Rasmussen and Kwon (2007) reported their work on Inquiry-Oriented Differential Equations (IO-DE) class, Inquiry-Oriented Instruction(IOI) has been widely used in the field in which researchers applied IOI in other content areas such as linear algebra (Wawro et al., 2012), scaled up curricular materials for IOI in abstract algebra courses (Larsen et al., 2013), and theorized principles for enacting IOI in practice (Kuster et al., 2018). IO-DE project exemplify a research-driven reform in instructional practices of excellence in mathematics education that have been led by the field of research in university mathematics education

Inquiry-Oriented Teacher Actions (IOTA) Project

In the past decades the K-16 mathematics education community has strived to improve the teaching and learning of mathematics via a concerted effort to develop innovative curriculum, to train more effective and knowledgeable teachers, to better understand how students build mathematical ideas, and to better understand how teachers create and sustain mathematics classrooms in which students learn mathematics in powerful and deep ways. Much progress has been made in terms of curriculum development and building models of students' mathematical learning. Much less progress has been made, however, in understanding how it is that teachers create and sustain classroom learning environments in which students build robust relational understandings of mathematics and develop desirable dispositions and attitudes towards knowing and doing mathematics. Indeed, past research as well as our experiences with undergraduate mathematics teachers demonstrates that it is quite difficult for teachers to develop and sustain such classroom learning environments. Models of how teachers accomplish this task would contribute both theoretically to the literature on teaching and practically to professional development efforts.

The goal of the *Inquiry-Oriented Teacher Actions* (IOTA) Project is to develop a model for how it is that teachers create and sustain inquiry-oriented learning environments in which students gain mathematical power and sophistication. In particular, we focus on characterizing teachers' discursive moves in inquiry-oriented classrooms. We use an innovative approach to differential equations, referred to as the IO-DE project as a case example.

We define *inquiry-oriented learning environments* as those classrooms that have two distinguishing features. First, regarding student activity, students routinely explain and justify their thinking and listen to and attempt to make sense of others' ideas. That is, students engage in genuine argumentation as they build mathematical ideas. Regarding teacher activity, teachers routinely inquire into how it is that students are thinking about the mathematics. In other words, teachers are continually attempting to understand their students' mathematical reasoning. Such understanding contributes to their decisions about how to proceed to advance their mathematical agenda.

As a start to define *discursive moves*, we operationalize discursive moves in terms of the following three types of teacher actions: Teacher Questioning, Teacher Revoicing, and Teacher Telling. We leave open the possibility that our analysis will reveal other types of discursive moves. In addition, these discursive moves are intended to include verbal utterances as well as their kinaesthetic actions, such as gestures.

Kwon et al. (2008) detail four different functions of the teacher's revoicing in an inquiry-oriented classroom, because it is one of the discursive strategies that often occurs in the teaching of mathematics, but which has received limited attention in mathematics education research at the undergraduate level. Our analysis shows that a teacher's revoicing can constitute a major repertoire of his or her discursive moves and carries out critical functions in the context of mathematics practice in class. For example, one function of revoicing identified was that of a binder – in which the teacher's revoicing created a context for students to bring up and align themselves with diverse mathematical positions – which supported the discursive, social process of negotiating meaning. Theoretically, these pedagogical moves were related to the instructional design theory of RME (Rasmussen & Kwon, 2007) and Vygotsky's notion of culture tool. Pragmatically, these moves provide strategies for others who wish to create mathematical discursive communities to support students' evolving mathematical reasoning.

A Community-Based Teacher Professional Development Model

Kwon et al. (2014) introduced a conceptual framework and practices, yield by research, into a teacher professional development program focusing on teacher community for mathematics teachers to increase professionalism. Conceptually, it was distinguished from the other training programs in terms of the participants, curriculum and methods. The teacher communities consisting of three or four teachers from the same school, as well as a mentor and sub-mentor, master, or professional teachers with professional expertise and executive capability. The curriculum of our program includes some process practicing and reflecting of teachers' communities on their own classes. The program's structure required active participation. Through our program, the teachers improved their teaching competency. Also, the operational ability of the teacher learning communities was improved. A teaching and learning community culture had been formed in each school, which showed that the community could continue even though the PD was no longer being conducted at the school operated even after our program was over. In the past, teachers avoided opening up their classrooms for others to observe, as this was previously regarded as a form of teacher evaluation in Korean classroom culture. However, the teachers who participated in the program now offered to open up their classrooms for other teaching community members, and saw this as an opportunity to contribute to improving the teaching competency of the community.

The ultimate purpose of the community-based mathematics teachers PD program that was developed by this research is to support continuous development of teachers' professionalism through training, where professionalism of mathematics teachers is regarded as a factor enhancing their ability to improve their lessons and help students' learning. To this end, rather than transferring all responsibilities to individual teachers, their professionalism was enhanced by growth through collaboration and reflection within the teachers' community.

The concept and procedural model of the training program developed by this research may be modified to suit the needs of course subjects other than mathematics, so that the model can be applied to the operation of PD programs for these other subjects. This systematic PD program will facilitate sustainable development of teachers' professionalism as teacher-researcher, the spread of community among teachers, and the enhancement of teachers' capability to implement the learning material, thereby creating positive change in mathematics education. In fact, inspired by these positive effects, the Korea Foundation for the Advancement of Science and Creativity (KOFAC) is implementing our PD program model in its PD program for elementary school teachers to foster mathematics classes based on storytelling.

Final Words

How can we inspire leaners to excel? To achieve excellent learning outcomes, we need excellent teachers. These projects discussed in this paper provide models towards excellence in teacher education. It is clear that these models need to be investigated in more depth, both as research topics and innovative practices.

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